

Integrator-Backstepping Based Control for Nonlinear Roll-to-Roll Web Dynamics

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Abstract: The paper presents a method for control analysis and design of roll-to-roll (R2R) dynamic system for printed electronics technology. The research provides an advanced control analysis and design methodology for a general multioutput multi-input (MIMO) strict-feedback class of nonlinear dynamic systems. The research starts with an integrator-backstepping control (IBSC) law formulation for the system by defining the modified tracking error in term of integral of normal tracking error. Considering each sublayer of control design process via a chosen Control Lyapunov function (CLF) and parameter assignment results in an IBSC law. Then, an IBSC law-based control strategy is provided for the design model. For the application of the proposed method, dynamic analysis on roll-to-roll (R2R) web system is made to transform a general nonlinear R2R web dynamics into a standard MIMO strict-feedback form for control analysis and design purposes. With the achieved model, an IBSC-based control algorithm is provided for numerical simulation and experiments. A control software is developed for implementing studies of numerical simulation and experiments to validate the reliability and feasibility of the proposed method

Keywords: *Integrator-backstepping, nonlinear control, roll-to-roll system*

INTRODUCTION

Roll-to-roll (R2R) manufacturing technology plays an important role in printed electronics in order to make devices at high speeds with lower cost. With the rapid development of mathematical tools and high-performance computers, investigations on control analysis and design for nonlinear R2R web systems are addressed by many researchers [1][2]. The standard approach for designing controllers for nonlinear dynamic systems is gain-scheduling. In this strategy, linear approximation of dynamic equations at several important operating points is achieved. Depending on these points, linear controllers are designed and then combined continuously from one operating point to another. Due to linearization, the actual system performance and stability can be significantly different from the design results due to the approximated nonlinearities in [1]. In recent years, control analysis and design of nonlinear dynamic systems are addressed by researchers [3]-[10]. A nonlinear observer-based control method is proposed in [3]-[4] in replacement of tension transducers (or loadcells) and estimation of friction and inertia of rewinder and unwinder. Achieving an accurate mathematical model [5]-[8] is one of the most important aspects in control analysis and design for R2R web dynamic system. In these

approaches, better assumptions compared to standard models are made to achieve more accurate models. Works in [9]-[14] propose control strategies on disturbance rejection. The works aim to reject these disturbances caused by unknown shapes of rolls. In order to reject the tension disturbance due to the roll shape, an adaptive eccentricity estimator and eccentricity compensation methods are proposed to improve performance [11]. Works in [2,15]-[18], developed by authors of this paper, present backstepping-based control methods to improve the precision and stability. However, the variation of the radius of rewinder and unwinder is not included in these works.

In this paper, an integrator is introduced in the analysis and design process for MIMO strict-feedback model of nonlinear dynamic systems, and then systematic steps are provided for formulating the IBSC law for the model. An IBSC based control algorithm is proposed for nonlinear two-span R2R web dynamic system with design parameters selected optimally by using the modified genetic algorithm [11]. Numerical simulation studies validate and compare to the work [17] in which a backstepping-based control without an integrator is implemented.

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EXPMATHEMATICAL BACKGROUND

In this section, a definition and theorem are provided to assist in developing the proposed control design for a strict-feedback system and a theorem on the convergence of integral-coordinate transformation is provided and proven to support the IBSC method. Then, a systematic procedure is presented for formulating the IBSC law for a MIMO strict-feedback system.

Concepts and Theorem

Consider a general equation of nonlinear dynamic systems as

$$\dot{x} = f(x) \tag{1}$$

Where $x, f \in \mathbb{R}^n$

Definition 1. (Control Lyapunov Function)

A function $V(x): \mathbb{R}^n \rightarrow \mathbb{R}$ is said to be control Lyapunov function (CLF) with respect to the system (1) if the Lie derivative of $V(x)$ with respect to $f(x)$ such that

$$\dot{V}(x) = \frac{\partial V(x)}{\partial x} f(x) = -W(x) < 0, \forall x \neq 0 \tag{2}$$

where $W(x)$ is a positive definite function.

Theorem 1. (Lyapunov Asymptotic Stability)

The system (1) is said to be asymptotically stable at an equilibrium state in the sense of Lyapunov if there exists a function $V(x): \mathbb{R}^n \rightarrow \mathbb{R}$ such that $V(x)$ is a positive definite, decrescent, and CLF.

Consider a coordinate transformation as

$$\zeta_1 = (x_1 - x_{ref}) + c_0 \int_0^t e(\tau) d\tau \tag{3}$$

Where ζ_1 is a state in new coordinate, x_1 is a state in original coordinate, x_{ref} is a constant, c_0 is a positive gain, t is a time variable and $e(t)$ is defined as normal tracking error.

Theorem 2: (Integral Convergence)

If the state ζ_1 in equation (3) tends to a zero value, then the corresponding state x_1 will converge to the command value x_{ref} for all positive gains c_0 as t goes to the infinity. Proof Assume the state ζ_1 tends to a zero value at a certain time t . Then, the equation (3) can be rewritten as

$$(x_1 - x_{ref}) + c_0 \int_0^t e(\tau) d\tau = 0 \tag{4}$$

By using the expression $e(t) = (x_1 - x_{ref})$, equation (4) can be re-written as

$$e(t) + c_0 \int_0^t e(\tau) d\tau = 0 \tag{5}$$

Thus, taking the derivative in time on both sides of the equation (5) yields

$$\dot{e}(t) + c_0 e(t) = 0 \tag{6}$$

By examining the system in equation (6), it is easy to show that the solution of equation (6) (or normal error $e(t)$ converges to zero with every positive gain c_0 as t

goes to infinity, implying the state x_1 converges to the command value x_{ref} .

INTEGRATOR-BACKSTEPPING CONTROL

The research starts with the concept of backstepping design for a MIMO strict-feedback model of nonlinear dynamic systems. A backstepping-based design is seen as a recursive design process which breaks a design problem on the full system down to a sequence of sub-problems on lower order systems. Considering each lower order system with a Control Lyapunov Function (CLF) and paying attention to the interaction between two subsystems makes it modular and easy to design the stabilizing controller. A MIMO strict-feedback system in Eqs. (7) - (10) is addressed for formulating the stabilizing control law but a general form of n th order can be extended similarly.

Consider a MIMO strict-feedback form of nonlinear dynamic system in [16].

$$\dot{x}_1 = f_1(x_1, x_2, x_3) \tag{7}$$

$$\dot{x}_2 = f_2(x_1, x_2, x_3, x_4) + g_1(x_1, x_2, x_3, x_4)u_1 \tag{8}$$

$$\dot{x}_3 = f_3(x_1, x_2, x_3, x_4) \tag{9}$$

$$\dot{x}_4 = f_4(x_1, x_2, x_3, x_4) + g_2(x_1, x_2, x_3, x_4)u_2 \tag{10}$$

where $x \in \mathbb{R}^4$, $f(x)$ and $g(x)$ valued functions, $u \in \mathbb{R}^2$, $y \in \mathbb{R}^2$, $h(x)$ is n -valued function.

It is assumed that the system has a well-defined relative degree at equilibrium points and the x_2, x_4 variables in Eqs. (7) and (9) are solvable explicitly in terms of the other variables. Thus, input-output decoupling problem is solvable by a state space feedback exact linearization law and the state feedback law is designed using the backstepping approach.

The objective is to design a control law for the nonlinear control system (7)-(10) such that $x_1 \rightarrow x_{1ref}$, and $x_3 \rightarrow x_{3ref}$ asymptotically where x_{1ref} and x_{3ref} are constant and globally asymptotical stability (GAS) in sense of the Lyapunov Theorem is achieved with no overshoots in the system. The following are the steps for formulating the IBSC laws for the system.

A backstepping control law for the strict-feedback model is formulated in principle in which the whole system is divided into n subsystems in a principle such that the i th subsystem consists of the $(i-1)$ th subsystem plus an extra state and the n th subsystem is the original n th order system via coordinate transformations and feedbacks. By applying consecutively, the coordinate transformation and choosing a feedback law via the Control Lyapunov Function to each subsystem from the lowest to highest order and rewriting the feedback law in the original coordinates, the resulting controllers make the original deficient system a well-tracking command and asymptotically global stable. The following steps are used for formulating the IBSC for the system in Eqs. (7) - (10).

Step 1: x_2 is regarded as a control input in Eq. (2) that is considered as the first subsystem. Thus, x_2 is chosen to make the first subsystem GAS. The choice is called a virtual control law, i.e. By putting

$$\zeta_1 = (x_1 - x_{ref}) + c_0\sigma(11)$$

Where $\sigma = \int_0^t e(\tau)d\tau$, $e(t) = (x_1 - x_{ref})$ and $e(t)$ is defined as normal tracking error and c_0 is a positive gain. By differentiating both sides of the Equation (8) in time and combining with the Equation (2), we have:

$$\dot{\zeta}_1 = \dot{x}_1 = f_1(c_0, x_{ref}, \zeta_1 + x_{1ref}, x_2, x_3) \quad (12)$$

For the Equation (9), a CLF $V(\zeta_1)$ can be chosen such that when the virtual control law is applied, its time derivative becomes negative definite, i.e. $V_1 = \frac{1}{2}\zeta_1^2$

By taking derivative in time of the CLF and combining with the Equation (9) results in;

$$\dot{V}_1 = \zeta_1\dot{\zeta}_1 = \zeta_1 f_1(c_0, x_{ref}, \zeta_1 + x_{1ref}, x_2, x_3, \sigma) \quad (13)$$

By satisfying the GAS condition in a sense of Lyapunov IBSC for the Equation (1), a virtual control law α_1 can be chosen as follows;

$$\begin{aligned} -c_1\zeta_1 &= f_1(c_0, x_{ref}, \zeta_1 + x_{1ref}, x_2, x_3, \sigma) \\ \rightarrow \alpha_1 &\equiv x_2 = \alpha_1(c_0, c_1, x_{ref}, x_1, x_3, \sigma) \end{aligned} \quad (14)$$

where c_1 is the positive gain. By doing so, we have:

$$\dot{V}_1 = \zeta_1\dot{\zeta}_1 = -c_1\zeta_1^2 < 0 \quad \forall \zeta_1 \neq 0 \quad (15)$$

Step 2: By choosing the state feedback (14) and a change of coordinate (11) and (16), or

$$\zeta_2 = x_2 - \alpha_1 \quad (16)$$

The second subsystem can be rewritten as follows:

$$\left. \begin{aligned} \dot{\zeta}_1 &= -c_1\zeta_1 \\ \dot{\zeta}_2 &= f_2(x_1, x_2, x_3) + g_1(x_1, x_2, x_3)u_1 - \dot{\alpha}_1 \end{aligned} \right\} \quad (17)$$

A CLF $V_2(\zeta_1, \zeta_2)$ can be chosen such that it makes the subsystem (17) GAS with the virtual control law, i.e.

$$V_2 = V_1 + \frac{1}{2}\zeta_2^2 \quad (18)$$

Taking the derivative of the Equation (18) in time and combining with the Equation (17) result in;

$$\begin{aligned} \dot{V}_2 &= -c_1\zeta_1^2 + \zeta_2(f_2(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, x_3, x_4) \\ &+ g_1(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, x_3, x_4)u_1 - \dot{\alpha}_1) \end{aligned} \quad (19)$$

To meet the GAS condition in the sense of Lyapunov for the Equation (19), a control law u_1 can be chosen such that

$$\begin{aligned} -c_2\zeta_2 &= f_2(\zeta_1, \zeta_2, x_3, x_4) + g_1(\zeta_1, \zeta_2, x_3, x_4)u_1 - \dot{\alpha}_1 \\ \rightarrow u_1 &= \lambda_2(c_0, c_1, c_2, x_{ref}, x_1, x_2, x_3, x_4, \sigma) \end{aligned} \quad (20)$$

where c_2 is the positive gain. By doing so, we have:

$$\dot{V}_2 = -c_1\zeta_1^2 - c_2\zeta_2^2 < 0 \quad \forall \zeta_1, \zeta_2 \neq 0 \quad (21)$$

Step 3: by choosing the state feedbacks (14) and (20) and a change of state transformations (11), (16), the third subsystem can be rewritten as follows:

$$\left. \begin{aligned} \dot{\zeta}_1 &= -c_1\zeta_1 \\ \dot{\zeta}_2 &= -c_2\zeta_2 \\ \dot{x}_3 &= f_3(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, x_3, x_4) \end{aligned} \right\} \quad (22)$$

By putting

$$\zeta_3 = (x_3 - x_{3ref}) + c_0\sigma_1 \quad (23)$$

Where $\sigma_1 = \int_0^t e_1(\tau)d\tau$, $e_1(t) = (x_3 - x_{3ref})$ and $e_1(t)$ is defined as normal tracking error and c_0 is a positive gain. The third subsystem can be rewritten as follows:

$$\left. \begin{aligned} \dot{\zeta}_1 &= -c_1\zeta_1 \\ \dot{\zeta}_2 &= -c_2\zeta_2 \\ \dot{\zeta}_3 &= f_3(c_0, c_1, x_{1ref}, x_{3ref}, x_4, \sigma_1) \end{aligned} \right\} \quad (24)$$

Now, x_4 is regarded as a control input in the subsystem (24). So, x_4 can be chosen to make the subsystem (24) GAS. A CLF $V_3(\zeta_1, \zeta_2, \zeta_3)$ can be chosen such that it makes the subsystem (24) GAS with the virtual control law, i.e.

$$V_3 = V_2 + \frac{1}{2}\zeta_3^2 \quad (25)$$

By taking the derivative of the Equation (25) in time and combining with the Equation (24) result in;

$$\dot{V}_3 = -c_1\zeta_1^2 - c_2\zeta_2^2 + \zeta_3 f_3(c_0, c_1, x_{1ref}, x_{3ref}, x_4, \sigma_1) \quad (26)$$

To satisfy the GAS condition in the sense of Lyapunov for the Equation (26), a virtual control law α_2 can be chosen such that

$$\begin{aligned} -c_3\zeta_3 &= f_3(c_0, c_1, x_{1ref}, x_{3ref}, x_4, \sigma_1) \\ \alpha_3 &\equiv \alpha_3(c_0, c_1, c_2, c_3, x_{1ref}, x_{3ref}, \zeta_1, \zeta_2, \zeta_3, \sigma, \sigma_1) \end{aligned} \quad (27)$$

where c_3 is the positive gain. By doing so, we have:

$$\dot{V}_3 = -c_1\zeta_1^2 - c_2\zeta_2^2 - c_3\zeta_3^2 < 0 \quad \forall \zeta_1, \zeta_2, \zeta_3 \neq 0 \quad (28)$$

Step 4: by choosing the state feedbacks (14) (20) and (27) and a change of state transformations (11) (16) and (23), the complete system can be rewritten as follows:

$$\left. \begin{aligned} \dot{\zeta}_1 &= -c_1\zeta_1 \\ \dot{\zeta}_2 &= -c_2\zeta_2 \\ \dot{\zeta}_3 &= -c_3\zeta_3 \\ \dot{x}_4 &= f_4(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, \zeta_3 + x_{3ref}, x_4) + \\ &g_2(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, \zeta_3 + x_{3ref}, x_4)u_2 \end{aligned} \right\} \quad (29)$$

By putting

$$\zeta_4 = x_4 - \alpha_3 \quad (30)$$

the complete system is rewrite as follows:

$$\left. \begin{aligned} \dot{\zeta}_1 &= -c_1\zeta_1 \\ \dot{\zeta}_2 &= -c_2\zeta_2 \\ \dot{\zeta}_3 &= -c_3\zeta_3 \\ \dot{\zeta}_4 &= f_4(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, \zeta_3 + x_{3ref}, \zeta_4 + \alpha_3) \\ &+ g_2(\zeta_1 + x_{1ref}, \zeta_2 + \alpha_1, \zeta_3 + x_{3ref}, \zeta_4 + \alpha_3)u_2 - \dot{\alpha}_2 \end{aligned} \right\} \quad (31)$$

A CLF $V_4(\zeta_1, \zeta_2, \zeta_3, \zeta_4)$ can be chosen such that it makes the subsystem (31) GAS with control law, i.e.

$$V_4 = V_3 + \frac{1}{2}\zeta_4^2 \quad (32)$$

By taking the derivative of the Equation (32) in time and combining with the Equation (31) Result in;

$$\dot{V}_4 = -c_1\zeta_1^2 - c_2\zeta_2^2 - c_3\zeta_3^2 + \zeta_4(f_4 + g_2u_2 - \dot{\alpha}_2) \quad (33)$$

To satisfy the GAS condition in a sense of Lyapunov IBSC for the Equation (1), a control law u_2 can be chosen such that

$$-c_4\zeta_4 = f_4(\zeta_1, \zeta_2, \zeta_3, \zeta_4) + g_1(\zeta_1, \zeta_2, \zeta_3, \zeta_4)u_2 - \dot{\alpha}_2$$

$$u_2 = \alpha_4(c_0, c_1, c_2, c_3, c_4, x_{1ref}, x_{3ref}, x_i, \sigma, \sigma_1) \quad (34)$$

where c_4 is the positive gain. By doing so, we have:
 $\dot{V}_4 = -c_1\zeta_1^2 - c_2\zeta_2^2 - c_3\zeta_3^2 - c_4\zeta_4^2 < 0 \forall \zeta_i \neq 0 \quad (34)$
 Thus, by choosing the state feedbacks (14) (20) (27) and (34) and a change of state transformations (11) (16), (23) and (30), the nonlinear control system (7) – (10) is transformed into the decoupled linear control system

$$\left. \begin{aligned} \dot{\zeta}_1 &= -c_1\zeta_1 \\ \dot{\zeta}_2 &= -c_2\zeta_2 \\ \dot{\zeta}_3 &= -c_3\zeta_3 \\ \dot{\zeta}_4 &= -c_4\zeta_4 \end{aligned} \right\} \quad (35)$$

By examining the system (35), the system is stable and converges to zero with positive gains and the response of the system has no overshoots. The desired settling time and rising time of system are obtained by tuning the gains. Thus, the stability and performance specifications on the system (7)-(10) are achieved with the IBSC. In summary, the IBSC for the system (7) – (10) is given as follows

$$\left. \begin{aligned} u_1 &= \alpha_2(c_0, c_1, c_2, c_3, x_{1ref}, x_{3ref}, x_i, \sigma) \\ u_2 &= \alpha_4(c_0, c_1, c_2, c_3, c_4, x_{1ref}, x_{3ref}, x_i, \sigma, \sigma_1) \end{aligned} \right\}$$

where c_0, c_1, c_2, c_3 and c_4 are positive gains that are determined optimally by using the modified genetic algorithm [13].

APPLICATION OF NONLINEAR R2R WEB SYSTEM DYNAMICS

Figure 1 shows two-span R2R web system that consists of unwinder, rewinder, infeed, dancer system, two loadcells, rollers, and web lateral control system. The idle rollers guide the moving web around loadcell in a fixed angle. In order to control the web tension at span 1 and span 2, motors at unwinder and rewinder are used with control torques and respectively and the infeed is used for the web velocity. The dancer system put at unwind side is to take up the slack during start-up and shutdown. On the other hand, two loadcells are used to feedback the web tension during the operating process and a web guide mechanism is used to control web lateral error. It is assumed that no web slippage occurs, the web has no permanent deformation due to applied tension, and the load cell and dancer dynamics is ignored.

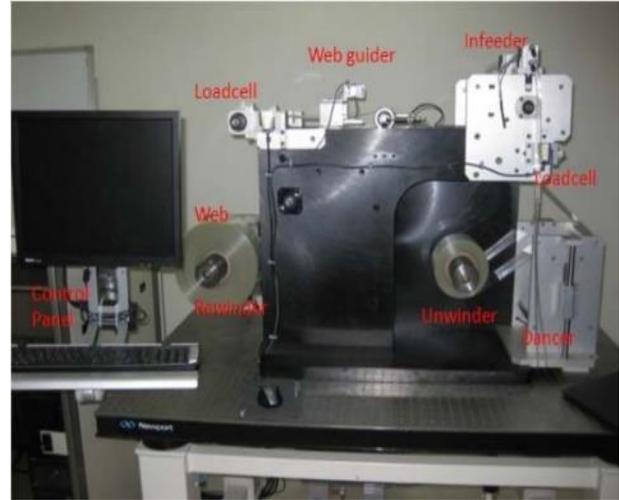


Figure 1. Two-Span Roll-to-Roll Web System

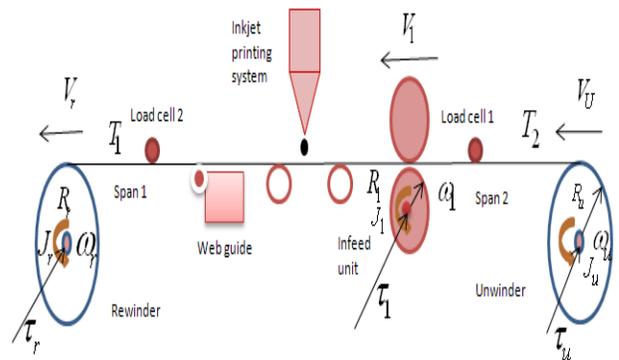


Figure 2 Model of Two-Span R2R Web System

By using Newton's law and the principle of mass conservation with assumptions, the non-linear dynamic equations of two-span roll-to-roll web control system can be written as follows.

$$\dot{\omega}_1 = k_7 T_2 + k_8 T_1 + k_9 \omega_1 + k_{10} \tau_1 \quad (36)$$

$$\dot{T}_1 = k_1 \omega_1 T_1 + k_2 \omega_1 + k_3 \omega_u \quad (37)$$

$$\dot{\omega}_u = k_4 T_1 + k_5 \omega_u + k_6 \tau_u \quad (38)$$

$$\dot{T}_2 = k_{11} \omega_1 T_1 + k_{12} \omega_r T_2 + k_{13} \omega_r + k_{14} \omega_1 \quad (39)$$

$$\dot{\omega}_r = k_{15} T_2 + k_{16} \omega_r + k_{17} \tau_r \quad (40)$$

Where $k_1 = -\frac{R_1}{L_1}, k_2 = \frac{EAR_1}{L_1}, k_3 = -\frac{EAR_u}{L_1}, k_4 =$

$$\frac{R_u}{J_u}, k_5 = -\frac{B_u}{J_u}, k_6 = \frac{1}{J_u}, k_7 = \frac{R_1}{J_1}, k_8 = -\frac{R_1}{J_1}, k_9 =$$

$$-\frac{B_1}{J_1}, k_{10} = \frac{1}{J_1}, k_{11} = \frac{R_1}{L_2}, k_{12} = -\frac{R_r}{L_2}, k_{13} =$$

$$\frac{EAR_r}{L_2}, k_{14} = -\frac{EAR_1}{L_2}, k_{15} = -\frac{R_r}{J_r}, k_{16} = -B_r/J_r, k_{17} = 1/J_r$$

T_1 = web tensions of span 1, T_2 = web tensions of span 2, ω_u = angular velocity of unwinder, ω_1 = angular velocity of infeed, ω_r = angular velocity of rewinder, J_u = inertia moment of unwinder and motor at unwinder, J_1 = inertia moment of roller 1 and motor at roller 1, J_r = inertia moment of winder and motor at winder, R_u = radius of unwinder, R_1 = radius of infeed roller 1, R_r = radii of

rewinder, L_1 = the length of span1, L_2 = the length of span 2, E: Young’s module of web materials, A: Area of cross-section, τ_u = control torque generated by the motor at unwinder, τ_1 = control torque generated by the motor at infeeder roller 1 , τ_r = control torque generated by the motor at rewinder , ρ = the density of the web, h = the thickness of web.

Problem Statement

The given problem is to design a nonlinear control law for control inputs τ_1, τ_u, τ_r for the system (36) - (40) so that web tensions T1 tracks T1ref and T2 tracks T2ref and angular velocity ω_1 tracks ω_{ref} asymptotically with zero or acceptable small overshoot in the presence of disturbance.

IBSC Law Formulation

The integrator-backstepping-based control design approach for the MIMO strict-feedback system is used for formulating an IBSC law τ_1, τ_u, τ_r for the system (36) - (40). In this approach, the complete system (36) - (40) is divided into five subsystems: the first subsystem consists of Eq. (36), the second subsystem consists of Eq. (36) in new state variable and Eq. (37) , the third subsystem consists of Eqs. (36), (37) in new state variables and Eq. (38), the fourth subsystem consists of Eqs. (36)-(38) in new state variables and Eq. (38), and the fifth subsystem (or complete system) consists of Eqs. (36)-(39) in new state variables and Eq. (40). Similar steps for IBSC law formulation for the system 36)-(40) in Section 2 are used for synthesizing the IBSC law τ_1, τ_u, τ_r . In this paper, such a detailed procedure is not presented for the concise purpose. By doing so, the IBSC law for the system (36) - (40) is achieved. With the achieved above controllers, a similar block diagram, as shown in s [16], [17], provides a control strategy for numerical simulation of two-span R2R web system in the next section.

Numerical Simulation Studies

Simulation Conditions: Data for R2R web system, as shown in Fig. 1, in [15], [16] are used for simulation implementation with the optimal gains in [14]. Numerical studies are implemented in two different operating profiles to show advantages of the proposed method. In Case 1, single-step command for web tensions $T1_{ref} = 20$ (N), $T2_{ref} = 15$ (N) and desired angular velocity on feeder $\omega_1 = 0.5$ rad/s is presented during 5 seconds without disturbance. In Case 2, same operating conditions as Case 1 is used for simulation during 30 seconds in the presence of varying radii of rewinder and unwinder in the second profile. Simulation studies center only on web tension control due to its importance in printed electronics technology. Thus, simulation results on web tension are shown and discussed as follows:

Simulation Result

Case 1: Single-Step Commands for 5 seconds

The blue line(or dash line) in Fig.3showsthe webtensionof span 1 and the red line (or solid line) in Fig. 3 shows the web tension of span 2. The results indicate the web tensions have a well-tracking behavior with no overshoot and asymptotical stability. This result is exactly coincided to the ones in [17] in which no integrator is added to the control process. The reason for this is that the radii of rewinder and unwinder are assumed to be constant, this is only true for a short period of time with slow web velocity. Thus, the numerical simulation is implemented for a long period of time as shown in Case 2.

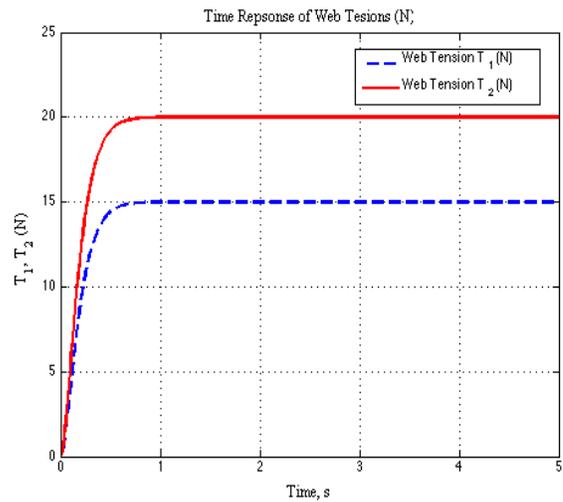


Figure 3 Time Response of Web Tensions of 5 seconds

Case 2: Single-Step Commands for 30 seconds

The blue line (or dash line) in Fig. 4 shows the web tension of span 1 without an integrator and the red line (or solid line) in Fig. 4 shows the web tension of span 2 without an integrator. The results indicate the web tensions have a divergence as time increases. Particularly, the web tension in span 1 increases as the radius of rewinder increases and the web tension in span 2 decreases as the radius of unwinder decreases. Similarly, the green line (or solid line) in Fig. 4 shows the web tension of span 1 with an integrator and the black line (or dot line) in Fig. 4 shows the web tension of span 2 with an integrator. Results show that the web tensions track asymptotically to the commands with stability as two integrators are used. This verifies the theoretical prediction, as shown in Theorem 2.

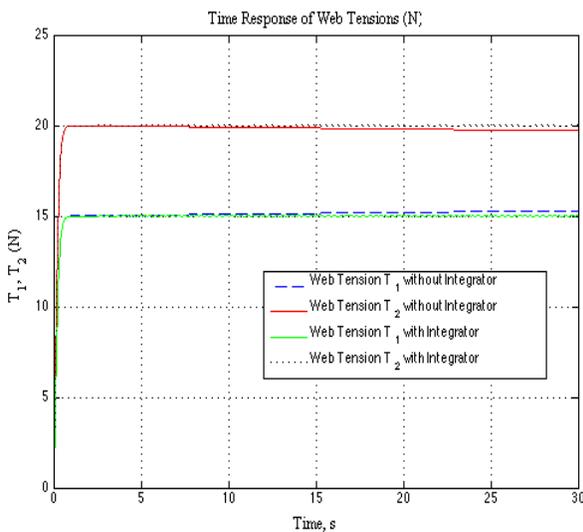


Figure 4 Time Response of Web Tensions of 30 seconds

CONCLUSION

In this paper, an IBSC-based control method is provided for a general MIMO strict-feedback model of nonlinear dynamic systems and then is applied for R2R web dynamic system. The IBSC-based control method can adapt and eliminate the varying parameters of R2R web system and provides with a robust control approach for varying-time parameter systems. Observing the simulation results and the above discussion, conclusions are made as follows: A systematic procedure is presented for formulating the IBSC law for a MIMO strict-feedback model; Application for R2R web system indicates that the IBSC-based control design achieves the performance specifications and stability; The IBSC-based control method provides a robust control strategy with a high accuracy in the presence of disturbance. With the rapid development of sensors and electronic devices, the IBSC-based control algorithm can result in a control system with high precision and are useful for applications with high digital computational system.

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