Journal of Engineering and Science Research 6 (6): 79-83, 2022
e-ISSN: 2289-7127

DOI: 10.26666/rmp.jesr.2022.6.7

Method of Line Solution of Optical Soliton in Optical Communication System

K.G.Tay[1,*], W.K. Tiong[2]

[1]Faculty of Electrical and Electronic Engineering,
Universiti Tun Hussein Onn Malaysia, Parit Raja, Batu Pahat,
86400 Johor, Malaysia,
[2]Universiti Malaysia Sarawak, 94300 Kota Samarahan, Sarawak, Malaysia
*Corresponding Author: tay@uthm.edu.my

Received: 01 November 2022; Revised: 05 November 2022; Accepted: 30 November 2022; Published: 30 December 2022

Abstract: The performance of fiber-optic communication is constrained by the dispersive phenomenon of group velocity dispersion (GVD) and fiber non-linearity of self-phase modulation, but when these effects are balanced, a single wave or soliton is produced. A soliton can travel a great distance with constant speed. It is suitable for use in ultra-long-distance communication because of these unique features. The non-linear Schrodinger (NLS) equation describes the optical soliton propagation in optical fibers. By using the method of line (MOL) to solve the NLS equation, this study mimics the optical soliton propagation. It demonstrates that during simulations at 400 km, the optical soliton maintains its shape and amplitude.

Keywords: *MOL, GVD, SPM, solitary wave*

1. Introduction

A fiber-optic telecommunication system is an answer to satisfy the demand for the ever-increasing internet speed. Optical light is sent into a fiber-optic transmission line which consists of a transmitter, an optical fiber as a channel and a receiver. Inevitably, optical fibers possess these characteristics: non-linearity called self-phase modulation (SPM), chromatic dispersion called group velocity dispersion (GVD) and fiber losses known as attenuation of the signals. Non-linearity and dispersion can lead to the distortion of the signal.

In fiber optics, GVD is related to the frequency-dependent fiber's refractive index. Therefore, pulses of distinct frequencies propagate through fiber optics at distinct speeds. As a result, they reach the destination at different times which causes temporal pulse spreading [1] and intersymbol interference [2-3]. This will later affect the functioning of the high bit rate in the long-haul telecommunication system. [2]

SPM is a nonlinear optical effect, which causes spectral broadening due to the phase shift induced by the wave itself. This phase shift is due to the refractive index of fiber, which is intensity-dependent. The front wave has a positive refractive index gradient, while the back wave has a negative refractive index gradient. The refractive index gradient reaches its maximum at the peak of the wave. This deviation of the refractive index gradient causes a phase shift and thus a change in the spectrum of a wave [1, 4].

A stable wave known as a soliton is formed when GVD and SPM are balanced. A soliton can travel in a dispersive medium undistorted for a long distance. As such, optical solitons are a promising candidate for ultra-long haul transmission and telecommunication [5].

The propagation of an optical pulse within an optical

Corresponding Author: K.G.Tay, Faculty of Electrical and Electronic Engineering, Universiti Tun Hussein Onn Malaysia, Parit Raja, Batu Pahat, 86400 Johor, Malaysia. Tel: +6074531427, Email tay@uthm.edu.my

fiber is described by the nonlinear Schrödinger (NLS) equation below.

$$\frac{\partial A}{\partial z} = -I\frac{\beta_2}{2}\frac{\partial^2 A}{\partial t^2} + I\gamma |A|^2 A - \frac{1}{2}\alpha A \qquad (1)$$

where **A** is the wave amplitude, I is an imaginary number, β_2 is GVD, γ is SPM and α is fiber loss parameters.

The "great wave of translation" in the Union Canal at Hermiston, which is close to the Riccarton campus of Heriot-Watt University, Edinburgh, was observed by a young Scottish engineer named John Scott Russell (1808-1882) while on horseback in August 1834 [6]. This incident is often credited with the discovery of soliton.

A well-known Dutch mathematician named Korteweg and his student de Vries came up with a non-linear evolution equation in 1895 for long, one-dimensional, surface gravity waves with small amplitudes that were moving through a shallow water channel. The Korteweg-de Vries (KdV) equation is the name given to this equation. The KdV equation's solution is the "great wave of translation" Rusell observed [7].

In 1965, Zabusky and Kruskal [8] used the finite-difference method to numerically solve the KdV equation. They gave this wave the name "soliton" because they observed that the properties of the KdV equation solution remained the same after the interaction. Zabusky and Kruskal were the first authors to use the term soliton.

In 1972, Zakharov and Shabat [9] solved the NLS equation by employing the inverse scattering method. which is an analytical approach. Nevertheless, the procedure entailed the application of a complex mathematical formulation to solve the NLS equation.

Hasegawa and Tapert [10] were the first to show theoretically and numerically the existence of soliton in an optical fiber in 1973 when there is a balance between SPM and GVD. However, it was only possible to use the soliton in real systems with the invention of the optical amplifier.

In 1980, Mollenauer, Stolen, and Golden [11] demonstrated the single-mode, 700-meter-long soliton transmission with a duration of 7 ps at a wavelength of 1550 nm in a silica-glass fiber.

Raman gain was suggested by Hasegawa [12] as an alternative to the repeater in optical communication. The Raman gain idea was demonstrated experimentally in by Mollenauer and Smith [13] in 1988 by transmitting 55ps soliton over 4000km of the all-optical transmission experiment.

After the Erbium-Doped Fiber Amplifier (EDFA) was created around 1989, a 25Gb/s soliton was successfully transmitted over 12,000km using a 75km fiber loop containing 3 EDFAs [6].

Research works [6], [11]-[13] show the potential of soliton application in long-haul communications.

2. Method of Line

Optical soliton has been simulated using the symmetrized split-step method and pseudospectral method in [14] and [15] respectively. However, these two methods require knowledge of the Fourier transform which is quite complicated. Here is a straightforward and efficient approach to solve the NLS equation: utilise the method of line (MOL) to simulate the propagation of an optical soliton.

The MOL approximates partial derivatives in partial differential equations (PDEs) using a 3-point central finite difference. This leads to a system of ordinary differential equations (ODEs). Then, this ODEs system can be solved using the classical techniques [3], [4] such as fourth-order Runge-Kutta (RK4).

The following 3-point central finite difference formulae are used to discretise the amplitude of the wave and temporal derivatives in (1) as follows:

$$A \approx \frac{A_{i+1} + A_i + A_{i-1}}{3}, \; A_{tt} \approx \frac{A_{i+1} - 2A_i + A_{i-1}}{(\Delta t)^2}. \qquad (2)$$

The index numbers i and Δt indicate the location and step size along t-axis, respectively. t-axis is split into N points for i = 0, 1, 2, ... , N−2, N−1. Thus, the MOL estimation of (1) is given by

$$\begin{aligned}\frac{dA_i}{dz} &= -I\frac{\beta_2}{2(\Delta t)^2}\left(A_{i-1} - 2A_i + A_{i+1}\right) + I\gamma |A_i|^2 A_i - \frac{1}{2}\alpha A_i \\ &\equiv f(A(t_i, z_j) = f(A_{i,j}).\end{aligned} \qquad (3)$$

Notice that Equation (3) consists of only one independent variable z, thus it is an ODE. Besides, it consists of N equations of ODEs as follows:

$$\begin{aligned}\frac{dA_0}{dz_j} &= -I\frac{\beta_2}{2(\Delta t)^2}\left(A_{-1,j} - 2A_{0,j} + A_{1,j}\right) + I\gamma |A_{0,j}|^2 A_{0,j} - \frac{1}{2}\alpha A_{0,j} \\ \frac{dA_1}{dz_j} &= -I\frac{\beta_2}{2(\Delta t)^2}\left(A_{0,j} - 2A_{1,j} + A_{2,j}\right) + I\gamma |A_{1,j}|^2 A_{1,j} - \frac{1}{2}\alpha A_{1,j} \\ \vdots &= \vdots \\ \frac{dA_{N-1}}{dz_j} &= -I\frac{\beta_2}{2(\Delta t)^2}\left(A_{N-2,j} - 2A_{N-1,j} + A_{N,j}\right) + I\gamma |A_{N-1,j}|^2 A_{N-1,j} - \frac{1}{2}\alpha A_{N-1,j}\end{aligned}$$

(4)

with the following initial condition

$$A(t_i, z=0) = A(t_i, 0) = A_{i,0}, \qquad i = 0, 1, 2, \cdots, N-2, N-1. \; (5)$$

Periodic boundary conditions below are applied to handle the point $A_{-1,j}$ and $A_{N,j}$

$$A_{-1,j} = A_{N-1,j}, A_{N,j} = A_{0,j} \qquad (6)$$

The system of N ODEs (4) is solved using the initial condition (5) and boundary conditions (6) until the desired distance of propagation with increment $\Delta z = 0.1$km.

Recall a system of 2 ODEs as follows:

$$\frac{dy_1}{dz} = f_1(z, y_1, y_2)$$
$$\frac{dy_2}{dz} = f_2(z, y_1, y_2) \quad (7)$$

with $y_1(z_0) = y_{1,0}, y_2(z_0) = y_{2,0}$. Its solution by the RK4 method at z_{j+1} will be given by

$$y_{i,j+1} = y_{i,j} + \frac{1}{6}(a_{i,j} + 2b_{i,j} + 2c_{i,j} + d_{i,j}), \quad i = 1, 2 \quad (8)$$

where

$$a_{i,j} = \Delta z f_i(z_j, y_{1,j}, y_{2,j})$$
$$b_{i,j} = \Delta z f_i\left(z_j + \frac{\Delta z}{2}, y_{1,j} + \frac{a_{1,j}}{2}, y_{2,j} + \frac{a_{2,j}}{2}\right)$$
$$c_{i,j} = \Delta z f_i\left(z_j + \frac{\Delta z}{2}, y_{1,j} + \frac{b_{1,j}}{2}, y_{2,j} + \frac{b_{2,j}}{2}\right) \quad (9)$$
$$d_{i,j} = \Delta z f_i(z_j + \Delta z, y_{1,j} + c_{1,j}, y_{2,j} + c_{2,j})$$

Similarly, when $j = 0$, using the initial condition $A_{i,0}$, RK4 is employed to solve the system of ODE (4) for the next space z_{j+1} to obtain $A_{i,1}$, $i = 0, 1, 2, \cdots, N-2, N-1$ as

$$A_{i,j+1} = A_{i,j} + \frac{1}{6}\left(a_{i,j} + 2b_{i,j} + 2c_{i,j} + d_{i,j}\right), \quad (5)$$

where

$$a_{i,j} = \Delta z f\left(A_{i,j}\right),\ b_{i,j} = \Delta z f\left(A_{i,j} + \frac{1}{2}a_{i,j}\right),$$
$$c_{i,j} = \Delta z f\left(A_{i,j} + \frac{1}{2}b_{i,j}\right),\ d_{i,j} = \Delta z f\left(A_{i,j} + c_{i,j}\right).$$

Here, Δz is an increment in the z-axis.

The equation is looped over until the desired distance of propagation with increment $\Delta z = 0.1$km. Notice that the last distance / Δz will give the number of iterations needed to iterate Eq.(5).

3 Results and Discussion

The MOL numerical scheme (3) is carried out using zero fiber loss, GVD value of -20ps^2/km, SPM value of 1.317 W^{-1}/km and initial solitary wave $A(t, z=0) = P_0 \operatorname{sech}\left(\frac{t}{T_0}\right)$, where P_0 = 0.30208W and $T_0 = 7.0902 \times 10^{-12}$ [16] for z up to 400km.

The initial wave for $-600ps \le t \le 600ps$ is depicted in Figure 1. The optical soliton propagates at one soliton period of 3.9482km using the above-mentioned parameters. Theoretically, when there is no fiber loss, the optical soliton can propagate undistorted for an unlimited distance. However, for graphical illustration, we can show a limited distance, in the present case 400km. Figures 2-5 show the 2D graphs at z = 100 km to z = 400 km respectively with an increment of 100km based on the MOL scheme whereas Figures 6-7 show the 3D simulation for z = 4km and z = 400km respectively. Figures 1-7. prove that the MOL can simulate the signal of optical fiber propagation. Non-linearity and dispersion constantly balance each other so the power is maintained inside the optical fiber and hence the pulse can travel in the optical fiber over a long distance ideally without broadening. This phenomenon provides a pulse that can propagate without distortion and broadening in the fiber optic.

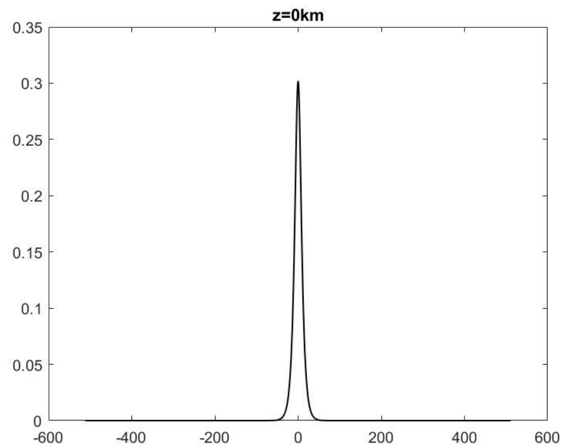

Fig. 1: Solitary wave at z =0km

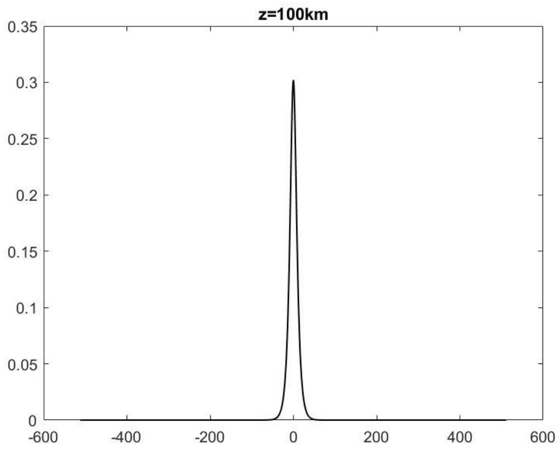

Fig. 2: Solitary wave at z = 100 km

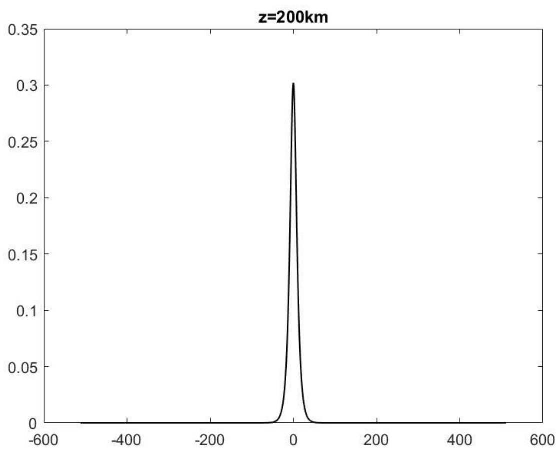

Fig. 3: Solitary wave at z = 200 km

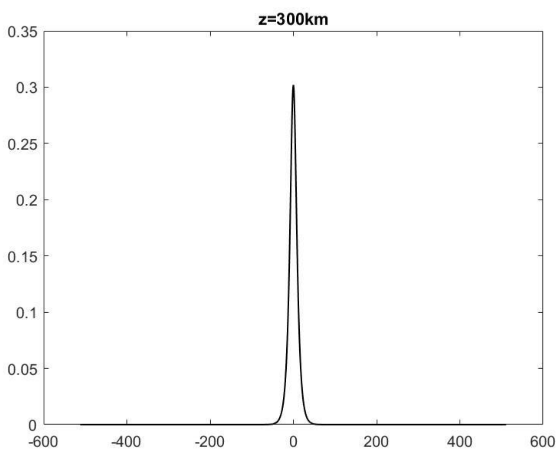

Fig. 4: Solitary wave at z = 300 km

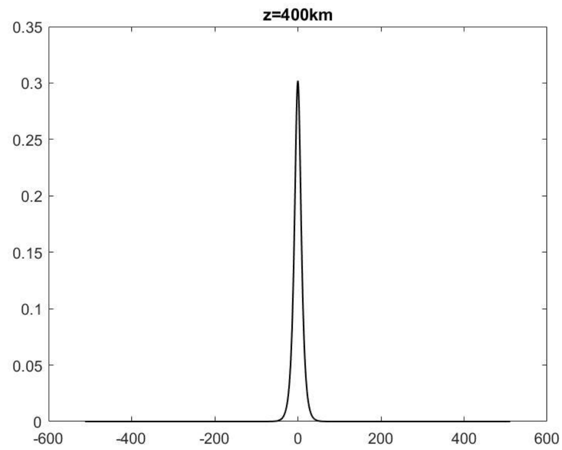

Fig. 5: Solitary wave at z = 400 km

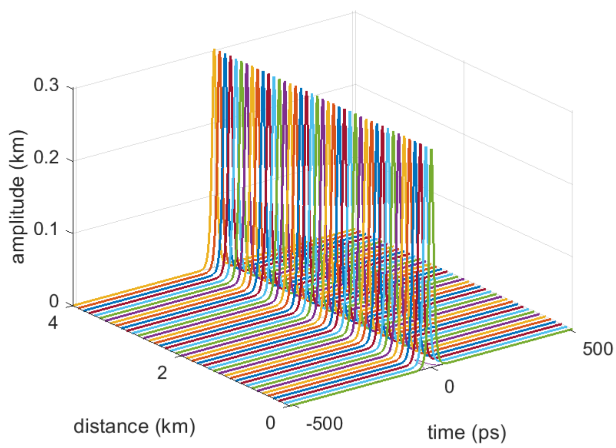

Fig. 6: 3D graph of soliton propagation for 40km

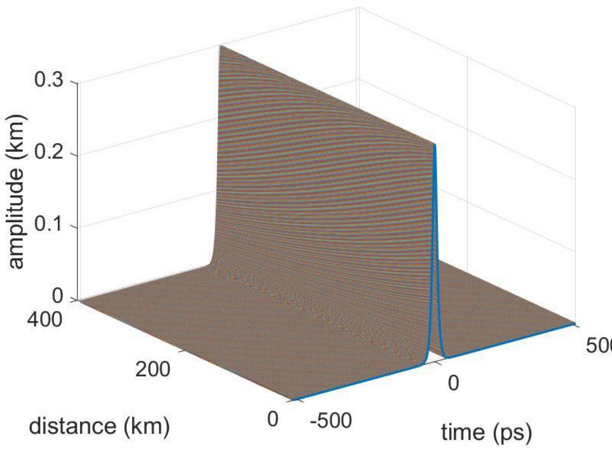

Fig. 7: 3D graph of soliton propagation for 400km

4. Conclusion

We performed an optical soliton simulation with the following parameters [16] ($\beta_2 = -20 ps^2 / km$, $\gamma = 1.317 W^{-1} km^{-1}$ and $\alpha = 0 dB / km$) using the method of line for 400km. It is shown that when there is an equilibrium between GVD and SPM, an optical solitary wave can propagate undistorted (shape and amplitude remain the same) for a long distance. With this distinctive feature, optical solitons are most suitable for application in long-haul optical communication.

5. Acknowledgements

This research was made possible by funding from Fundamental Research Grant Scheme number FRGS/1/2020/STG06/UTHM/03/7 provided by the Ministry of Higher Education, Malaysia.

6. References

[1] K. G. Tay, H. K. C. Audrey, W. S. Loi. Optical Soliton Simulation in Optical Fibers by OptiSystem, IOP Conf. Series: Materials Science and Engineering, Vol. 226,1-12, 2017.

[2] M.G. Patel and Khant S.B. Soliton transmission in fiber optic for long distance communication, International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol.3, No.2, 7100-7107, 2014.

[3] S. S Nair and V. Prem. Solitons: a promising technology in optical communication. International Journal of Science and Research (IJSR), Vol.3, No.1, 151-154, 2014.

[4] R. Gangwar, S. P. Singh and N. Singh. Soliton-Based Optical Communication, Progress in Electromagnetics Research, PIER. Vol.**74,** 157–166, 2007.

[5] R. L. Sharma and R. Singh. Solitons, its evolution and applicaltions in high speed optical communication, International Journal on Emerging Technologies, Vol. 2, No.2, 141-145, 2011.

[6] R. Kumar and M. Tiwari. Soliton in Optical Communication, 2005.

[7] H. A Hauss. Optical Fiber Solitons, Their Properties and Uses, Proceedings of the IEEE, Vol. 81, No.7, 970-983, 1983.

[8] N. J. Zabusky and M. D. Kruskal. Interaction of Solitons in a collisionless plasma and the recurrence of initial state, Physical Review Letters, Vol.15,240-243, 1965.

[9] V. F. Zakharov and A. B. Shabat. Exact Theory of Two-Dimensional Self-Focusing and One dimensional Self-Modulation of Waves, In Nonlinear Media Soviet Physics JETP, Vol.34, No.1, 62-69, 1972.

[10] A. Hasegawa and F. Tappert. Transmission of Stationary Nonlinear Optical Pulses in Dispersive Dielectric Fibers. I. Anomalous Dispersion, Appl. Phys. Lett., Vol.23,142-144, 1973.

[11] F. Mollenauer, L. R. H. Stolen and J. P. Gordon. Experimental Observation of Picosecond Pulse Narrowing and Soliton in Optical Fiber, Phys. Rev. Lett., Vol. 45, 1095-1098, 1980.

[12] A. Hasegawa. Amplification and reshaping of optical solitons in a glass fiber—IV: Use of the stimulated Raman process, Opt. Lett., Vol.8, 650–652, 1983.

[13] L. F. Mollenauer and K. Smith. Demonstration of soliton transmission over more than 4,000 km in fiber with loss periodically compensated by Raman gain, Opt. Lett. Vol.13, 675–677, 1988.

[14] K. G Tay, W. K. Tiong and Y. Y. Choy. Optical Soliton Simulation by Symmetrized Split Step Method, International Journal of Trends in Scientific Research and Development, 2(6): 175-180, 2018.

[15] K. G. Tay,, W. K. Tiong , W. S. Loi , C. T. Ong. Optical Soliton Simulation by Pseudospectral Method, Communications In Computational And Applied Mathematics, Vol.2. No.2, 17-21, 2020.

[16] _2008. OptiSystem Tutorials Communication System Design Software.Canada: Opti Wave.